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САМОСТОЯТЕЛЬНАЯ РАБОТА № 1

Тема: **Уравнения в частных производных 1-го порядка (часть 2)**

Вариант 1

1. Решите уравнения:

$$1) \quad xz \frac{\partial u}{\partial x} + yz \frac{\partial u}{\partial y} + xye^{z^2} \frac{\partial u}{\partial z} = 0;$$

$$2) \quad (x + 2u - u^2) \frac{\partial u}{\partial x} + (y + 3u^2 - u^3) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \quad 2ye^{y^2} \frac{\partial u}{\partial x} + \cos x \frac{\partial u}{\partial y} = 0, \quad u(0, y) = e^{y^2};$$

$$2) \quad x \frac{\partial u}{\partial x} = u, \quad u(2, y) = \sin y.$$

Вариант 2

1. Решите уравнения:

$$1) \quad \frac{xz}{3} \frac{\partial u}{\partial x} + yz \frac{\partial u}{\partial y} + x^3y(z^2 + 1)^2 \frac{\partial u}{\partial z} = 0;$$

$$2) \quad (x + e^{2u}) \frac{\partial u}{\partial x} + (y + 2e^{3u}) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \quad 4y \frac{\partial u}{\partial x} + e^x \frac{\partial u}{\partial y} = 0, \quad u(x, 0) = \sin e^x;$$

$$2) \quad x \frac{\partial u}{\partial y} = u^2, \quad u(x, 0) = x.$$

Вариант 3

1. Решите уравнения:

$$1) ye^{x^2+z} \frac{\partial u}{\partial x} + xy \frac{\partial u}{\partial y} + x \frac{\partial u}{\partial z} = 0;$$

$$2) (x-1) \frac{\partial u}{\partial x} + (y+1-u) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) 6y^2 \frac{\partial u}{\partial x} + \operatorname{ch} x \frac{\partial u}{\partial y} = 0, \quad u(0, y) = 8y^9;$$

$$2) e^x \frac{\partial u}{\partial y} = u^2, \quad u(x, 1) = e^x.$$

Вариант 4

1. Решите уравнения:

$$1) (x-2z) \frac{\partial u}{\partial x} + (y-2z) \frac{\partial u}{\partial y} + 3z \frac{\partial u}{\partial z} = 0;$$

$$2) (x+u-1) \frac{\partial u}{\partial x} + (y-u) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \sqrt{y} \frac{\partial u}{\partial x} + e^x \frac{\partial u}{\partial y} = 0, \quad u(x, 1) = e^x - 1;$$

$$2) \frac{\partial u}{\partial y} = u^2 \cos x, \quad u(x, 0) = \frac{2}{\sin 2x}.$$

Вариант 5

1. Решите уравнения:

$$1) \quad xz \frac{\partial u}{\partial x} + yz \frac{\partial u}{\partial y} + xy(z^2 + 1) \frac{\partial u}{\partial z} = 0;$$

$$2) \quad (2e^u - x) \frac{\partial u}{\partial x} + (3e^{2u} - y) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \quad x \cos y \frac{\partial u}{\partial x} - 2 \sin y \frac{\partial u}{\partial y} = 0, \quad u(1, y) = \sin^2 y;$$

$$2) \quad x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u, \quad u(x, 1) = x.$$

Вариант 6

1. Решите уравнения:

$$1) \quad \frac{xz}{2} \frac{\partial u}{\partial x} + yz \frac{\partial u}{\partial y} + x^2y(z^4 + 1) \frac{\partial u}{\partial z} = 0;$$

$$2) \quad (e^u - x) \frac{\partial u}{\partial x} + (u^2 + 2u - y) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \quad x \sin 2y \frac{\partial u}{\partial x} + \cos^2 y \frac{\partial u}{\partial y} = 0, \quad u(x, \pi) = x^2;$$

$$2) \quad x^2 \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} = 0, \quad u(x, -2) = x.$$

Вариант 7

1. Решите уравнения:

$$1) \frac{xz}{3} \frac{\partial u}{\partial x} + yz \frac{\partial u}{\partial y} + x^3 y (z^4 + 1) \frac{\partial u}{\partial z} = 0;$$

$$2) (u - x + 2) \frac{\partial u}{\partial x} + (2u - y + 1) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \operatorname{ch} x \sin y \frac{\partial u}{\partial x} + \operatorname{sh} x \cos y \frac{\partial u}{\partial y} = 0, \quad u(x, \pi) = -\operatorname{ch} x;$$

$$2) u \frac{\partial u}{\partial x} + e^{-y} \frac{\partial u}{\partial y} = 0, \quad u(x, \ln 2) = x.$$

Вариант 8

1. Решите уравнения:

$$1) xy \frac{\partial u}{\partial x} + xe^z \sqrt{1-y^4} \frac{\partial u}{\partial y} + y \frac{\partial u}{\partial z} = 0;$$

$$2) (\cos u + \sin u - x) \frac{\partial u}{\partial x} - (\cos u + \sin u + y) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) 3x \cos y \frac{\partial u}{\partial x} - \sin y \frac{\partial u}{\partial y} = 0, \quad u\left(x, \frac{\pi}{2}\right) = \sin x;$$

$$2) e^x \frac{\partial u}{\partial x} - u \frac{\partial u}{\partial y} = 0, \quad u(0, y) = y + 1.$$

Вариант 9

1. Решите уравнения:

$$1) \frac{xz}{2} \frac{\partial u}{\partial x} + z \operatorname{tg} y \frac{\partial u}{\partial y} + x \sin y \frac{\partial u}{\partial z} = 0;$$

$$2) (\cos u - \sin u - x) \frac{\partial u}{\partial x} + (\sin u - \cos u - y) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) (1 + y^2)^{-1} \operatorname{arctg} y \frac{\partial u}{\partial y} - x \frac{\partial u}{\partial x} = 0, \quad u(1, y) = \operatorname{arctg}^2 y;$$

$$2) u \frac{\partial u}{\partial x} - y^2 \frac{\partial u}{\partial y} = 0, \quad u(x, 1) = x + 1.$$

Вариант 10

1. Решите уравнения:

$$1) yz \operatorname{ctg} x^2 \frac{\partial u}{\partial x} + xy \frac{\partial u}{\partial y} + xz \frac{\partial u}{\partial z} = 0;$$

$$2) 2u(u^2 - x) \frac{\partial u}{\partial x} + (u^3 + 3u^2 - y) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \frac{\partial u}{\partial x} - \operatorname{arctg} x \frac{\partial u}{\partial y} = 0, \quad u(0, y) = y^2;$$

$$2) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u, \quad u(x, 1) = x^2.$$

Вариант 11

1. Решите уравнения:

$$1) \frac{xy}{2} \frac{\partial u}{\partial x} + x^2 z (y^2 + 1) \frac{\partial u}{\partial y} + yz \frac{\partial u}{\partial z} = 0;$$

$$2) (2 \cos u - \sin u - 2x) \frac{\partial u}{\partial x} + (2 \sin u + \cos u - 2y) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) 2 \ln y \frac{\partial u}{\partial x} + y \operatorname{tg}^2 x \frac{\partial u}{\partial y} = 0, \quad u(\pi, y) = (\pi + \ln^2 y)^2;$$

$$2) x \frac{\partial u}{\partial x} + 2y \frac{\partial u}{\partial y} = 4u, \quad u(1, y) = y.$$

Вариант 12

1. Решите уравнения:

$$1) xz \frac{\partial u}{\partial x} + z \frac{\partial u}{\partial y} + xe^y (z^2 + 1) \frac{\partial u}{\partial z} = 0;$$

$$2) (3e^u - 2x) \frac{\partial u}{\partial x} + (e^{-u} - 2y) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \sin y \frac{\partial u}{\partial x} + 3 \sin^2 x \cos^2 y \cos x \frac{\partial u}{\partial y} = 0, \quad u(0, y) = \cos y;$$

$$2) 3x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 6u, \quad u(x, 1) = x^3.$$

Вариант 13

1. Решите уравнения:

$$1) \quad xz \frac{\partial u}{\partial x} + z \operatorname{tg} y \frac{\partial u}{\partial y} + x \sin y \frac{\partial u}{\partial z} = 0;$$

$$2) \quad 2(u + u^2 - x) \frac{\partial u}{\partial x} - 2(u + u^2 + y) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \quad \sqrt{xy} \ln y \frac{\partial u}{\partial x} + e^{\sqrt{x}} \frac{\partial u}{\partial y} = 0, \quad u(x, 1) = 2e^{\sqrt{x}};$$

$$2) \quad x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = 2u, \quad u(2, y) = \sin y.$$

Вариант 14

1. Решите уравнения:

$$1) \quad xy \frac{\partial u}{\partial x} - xz(y^2 + 1)^2 \frac{\partial u}{\partial y} + yz \frac{\partial u}{\partial z} = 0;$$

$$2) \quad (4 + 3u - 3x) \frac{\partial u}{\partial x} + (6u - 1 - 3y) \frac{\partial u}{\partial y} = 1.$$

2. Решите задачи Коши:

$$1) \quad (x^2 + 1) \frac{\partial u}{\partial x} + \sqrt{y} \frac{\partial u}{\partial y} = 0, \quad u(0, y) = \sqrt{y};$$

$$2) \quad x \frac{\partial u}{\partial x} + 5y \frac{\partial u}{\partial y} = 10u, \quad u(-1, y) = y^3.$$
