1)
$$\begin{cases} dy - (1+y^2)\,dx = 0 \\ dy + (1+y^2)\,dx = 0 \end{cases} \qquad \frac{dy}{1+y^2} - dx = 0$$

$$u(x,y) = v(\xi, \eta)$$

$$u_y = v_\xi \cdot \xi_y + v_\eta \cdot \eta_y$$

$$(\cdot)_y = (\cdot)_\xi \cdot \xi_y + (\cdot)_\eta\, \eta_y$$

2)
$$u_y = \frac{1}{1+y^2} v_\xi - \frac{1}{1+y^2} v_\eta$$

$$u_{yy} = (u_y)_y = \frac{1}{(1+y^2)^2} v_{\xi\xi} - \frac{2}{(1+y^2)^2} v_{\xi\eta} + \frac{1}{(1+y^2)^2} v_{\eta\eta} + v_\xi \cdot \left(\frac{1}{1+y^2}\right)' - v_\eta \cdot \left(\frac{1}{1+y^2}\right)'$$

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$$\mathcal{D} = (1+y^2)^2 > 0 \quad \forall x, y \in \mathbb{R}$$

$$\mathcal{D} = (x^2+1)y^2 \geq 0, \quad 1)\ y = 0 \longrightarrow$$

$$2)\ y \neq 0 \longrightarrow$$

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$$u_{xx} + 4u_{xy} - 5u_{yy} + u_x - u_y = 0$$

$$u(x,0) = 2x, \quad u_y(x,0) = 1$$

1. $A = 1, \ B = 2, \ C = -5, \qquad B^2 - AC = \underline{\underline{9}} > 0 \longrightarrow \Gamma$

2. $(dy)^2 - 4\,dy\,dx - 5(dx)^2 = 0$:

$$(dy)^2 - 4\,dy\,dx + 4(dx)^2 - 9(dx)^2 = 0$$

$$(dy - 2dx)^2 - (3dx)^2 = 0$$

$$(dy - 5dx)(dy + dx) = 0$$

$$\left\{\begin{array}{l} dy - 5dx = 0 \\ dy + dx = 0 \end{array}\right. \qquad \left[\begin{array}{l} y - 5x = C_1 \\ y + x = C_2 \end{array}\right.$$

3. $\left\{\begin{array}{l} \xi = y - 5x \\ \eta = y + x \end{array}\right.$

4. $u(x,y) = v(\overbrace{y - 5x}^{\xi}, \overbrace{y + x}^{\eta})$

$$\begin{array}{r|l} 1 & u_x = -5v_\xi + v_\eta \\ -1 & u_y = v_\xi + v_\eta \\ 1 & u_{xx} = 25v_{\xi\xi} - 10v_{\xi\eta} + v_{\eta\eta}, \\ -5 & u_{yy} = v_{\xi\xi} + 2v_{\xi\eta} + v_{\eta\eta} \\ 4 & u_{xy} = -5v_{\xi\xi} - 4v_{\xi\eta} + v_{\eta\eta} \end{array}$$

$v_{\xi\xi}$: $25 - 5 - 20 = 0$

$v_{\eta\eta}$: $1 - 5 + 4 = 0$

$v_{\xi\eta}$: $-10 - 10 - 16 = -36$

v_ξ: $-5 - 1 = -6$

v_η: $1 - 1 = 0$

$$-36\,v_{\xi\eta} - 6\,v_\xi = 0$$

$$\underline{v_{\xi\eta} + \frac{1}{6}v_\xi = 0}$$

4. $v_{\xi\eta} + \frac{1}{6} v_\xi = 0$

$$\begin{cases} v_\xi = w \\ w_\eta + \frac{1}{6} w = 0 \end{cases}$$

$v_\eta + \frac{1}{6} v = C(\eta)$

$w_\eta + \frac{1}{6} w = 0$

$\lambda + \frac{1}{6} = 0 \qquad e^{-\frac{\eta}{6}}$

$w = C(\xi) e^{-\frac{\eta}{6}}, \quad v_\xi = C(\xi) e^{-\eta/6}, \quad v(\xi,\eta) = \underbrace{\int C(\xi) e^{-\eta/6} d\xi}_{e^{-\eta/6} \underbrace{\int C(\xi) d\xi}_{\Phi(\xi)}} + F(\eta)$

$$v(\xi,\eta) = e^{-\eta/6} \cdot \Phi(\xi) + F(\eta)$$

5. $u(x,y) = e^{-\frac{x+y}{6}} \Phi(y-5x) + F(x+y) \qquad (*)$

$u(x,0) = 2x$
$u_y(x,0) = 1$

6. $u_y(x,y) = -\frac{1}{6} e^{-\frac{x+y}{6}} \cdot \Phi(y-5x) + e^{-\frac{x+y}{6}} \cdot \Phi'(y-5x) + F'(x+y) \qquad (**)$

$$\begin{cases} u(x,0) \overset{(*)}{=} e^{-\frac{x}{6}} \cdot \Phi(-5x) + F(x) = 2x \\ u_y(x,0) \overset{(+x)}{=} -\frac{1}{6} e^{-\frac{x}{6}} \cdot \Phi(-5x) + e^{-\frac{x}{6}} \Phi'(-5x) + F'(x) = 1 \end{cases}$$

$$F(x) = 2x - e^{-x/6} \Phi(-5x) \quad V$$

$$\underbrace{-\frac{1}{6} e^{-\frac{x}{6}} \Phi(-5x)} + e^{-\frac{x}{6}} \Phi'(-5x) + 2 + \underbrace{\frac{1}{6} e^{-x/6} \Phi(-5x)} + 5 e^{-x/6} \cdot \Phi'(-5x) = 1$$

$$6 e^{-\frac{x}{6}} \Phi'(-5x) = -1$$

$$\Phi'(\underbrace{-5x}_{t}) = -\frac{1}{6} e^{x/6}$$

$$\Phi'(t) = -\frac{1}{6} e^{-\frac{t}{30}}$$

$$\boxed{\Phi(t) = 5 e^{-t/30} + C} \rightarrow V$$

$$\Phi'(\varphi(x)) = f(x)$$

$$\varphi'(x) \Phi'(\varphi(x)) = \varphi'(x) f(x)$$

$$\int \varphi'(x) \Phi'(\varphi(x)) dx = \int \varphi'(x) f(x) dx + C$$

$$\int \Phi'(\varphi(x)) d\varphi(x) =$$

$$-5 \Phi'(-5x) = \frac{5}{6} e^{x/6}$$

$$\Phi(\underbrace{-5x}_{t}) = 5 e^{x/6} + C$$

$$\Phi(t) = 5 e^{-t/30} + C$$

$$F(x) = 2x - e^{-x/6} \cdot (5 e^{x/6} + C) =$$

$$= \underline{\underline{2x - 5 - C \cdot e^{-x/6}}}$$

$\Phi, F \longrightarrow (*)$

$$\underline{u(x,y)} = e^{-\frac{x+y}{6}} \cdot \left(5 \cdot e^{-\frac{y-5x}{30}} + C\right) + 2(x+y) - 5 - C \cdot e^{-\frac{x+y}{6}} = \underline{2(y+x) - 5e^{-\frac{y}{5}} - 5}$$

с. 86

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№ 40(3) $\longrightarrow$ кан., общ. реш. | з. К