



[П] Проскуряков И.В. **Сборник задач по линейной алгебре.** – СПб.: Издательство «Лань», 2010. URL: http://elibrary.sgu.ru/uch_lit/560.pdf

[Ф] Фаддеев Д.К., Соминский И.С. **Сборник задач по высшей алгебре.**
<http://bookre.org/reader?file=635343>

17.11.2021

Занятие № 10

Обратная матрица

№ 836

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$|A| = 4 - 6 = -2 \neq 0 \quad \exists! A^{-1}$$

1.

$$\begin{aligned} A_{11} &= 4 \\ A_{12} &= -3 \\ A_{21} &= -2 \\ A_{22} &= 1 \end{aligned}$$

$$A^* = \begin{pmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot A^* = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix} \quad \left[A \cdot A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]$$

$$\begin{array}{c}
 2 \\
 \sim \left(\underbrace{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}}_A \mid \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \cdot (-3I) \sim \left(\begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} \mid \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix} \right) \cdot \begin{array}{l} +II \\ \cdot (-\frac{1}{2}) \end{array} \sim \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mid \underbrace{\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}}_{A^{-1}} \right)
 \end{array}$$

~ 841

$$A = \begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix} \quad A^{-1} = ?$$

$$\left(\begin{pmatrix} 3 & -4 & 5 & \mid & 1 & 0 & 0 \\ 2 & -3 & 1 & \mid & 0 & 1 & 0 \\ 3 & -5 & -1 & \mid & 0 & 0 & 1 \end{pmatrix} \right) \cdot \begin{array}{l} -II \\ -I \end{array} \sim \left(\begin{pmatrix} 1 & -1 & 4 & \mid & 1 & -1 & 0 \\ 2 & -3 & 1 & \mid & 0 & 1 & 0 \\ 0 & -1 & -6 & \mid & -1 & 0 & 1 \end{pmatrix} \right) \cdot \begin{array}{l} -III \\ -2I \end{array} \sim \left(\begin{pmatrix} 1 & 0 & 10 & \mid & 2 & -1 & -1 \\ 0 & -1 & -7 & \mid & -2 & 3 & 0 \\ 0 & -1 & -6 & \mid & -1 & 0 & 1 \end{pmatrix} \right) \cdot \begin{array}{l} -II \\ -II \end{array} \sim$$

$$\sim \left(\begin{pmatrix} 1 & 0 & 10 & \mid & 2 & -1 & -1 \\ 0 & -1 & -7 & \mid & -2 & 3 & 0 \\ 0 & 0 & 1 & \mid & 1 & -3 & 1 \end{pmatrix} \right) \cdot \begin{array}{l} -10III \\ +7III \end{array} \sim \left(\begin{pmatrix} 1 & 0 & 0 & \mid & -8 & 29 & -11 \\ 0 & -1 & 0 & \mid & 5 & -18 & 7 \\ 0 & 0 & 1 & \mid & 1 & -3 & 1 \end{pmatrix} \right) \cdot (-I) \sim \left(\begin{pmatrix} 1 & 0 & 0 & \mid & -8 & 29 & -11 \\ 0 & 1 & 0 & \mid & 5 & -18 & 7 \\ 0 & 0 & 1 & \mid & 1 & -3 & 1 \end{pmatrix} \right)$$

$$A^{-1} = \begin{pmatrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{pmatrix}$$

№ 844

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \quad A^{-1} = ?$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} +II + III + IV \\ -I \\ -I \\ -I \end{array} \sim \left(\begin{array}{cccc|cccc} 4 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -2 & -1 & 0 & 1 & 0 \\ 0 & -2 & -2 & 0 & -1 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \\ \\ -III \\ -III \end{array} \sim \left(\begin{array}{cccc|cccc} 4 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 & -1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -2 & -1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 & 0 & 0 & -1 & 1 \end{array} \right) \begin{array}{l} \\ \\ -III \\ -III \end{array}$$

$$\sim \left(\begin{array}{cccc|cccc} 4 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 & -1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -2 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 & -1 & -1 & 1 \end{array} \right) \begin{array}{l} \\ +\frac{1}{2}IV \\ +\frac{1}{2}IV \\ \end{array} \sim \left(\begin{array}{cccc|cccc} 4 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & 0 & -1/2 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & -2 & 0 & 1/2 & 1/2 & -1/2 & 1/2 \\ 0 & 0 & 0 & 4 & 1 & -1 & -1 & 1 \end{array} \right) \begin{array}{l} \cdot 1/4 \\ \cdot (-1/2) \\ \cdot (-1/2) \\ \cdot 1/4 \end{array}$$

$$A^{-1} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & -1/4 & -1/4 \\ -1/4 & -1/4 & 1/4 & 1/4 \\ 1/4 & -1/4 & -1/4 & 1/4 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} = \frac{1}{4} A$$

Матричные уравнения

№ 861.

$$\begin{matrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix} \\ \underset{\text{A}}{\parallel} \qquad \qquad \qquad \underset{\text{B}}{\parallel} \end{matrix}$$

Если $|A| \neq 0$, то $\exists A^{-1}$

$$AX = B \quad A^{-1}(AX) = A^{-1}B, \quad (\underbrace{A^{-1}A}_{\parallel E})X = A^{-1}B, \quad \underline{X = A^{-1}B}$$

$$\text{№ 836} \rightarrow A^{-1} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$$

$$X = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix}, \quad AX = B, \quad X [3 \times 2]$$

$$X = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \end{pmatrix}$$

$$\begin{cases} x_{11} - x_{21} + 2x_{31} = 1 \\ x_{21} + 3x_{31} = -1 \\ \hline x_{12} - x_{22} + 2x_{32} = 2 \\ x_{22} + 3x_{32} = 2 \end{cases}$$

$$\left(\begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 2 \\ 0 & 1 & 3 & -1 & 2 \end{array} \right)^{+II} \sim \left(\begin{array}{ccc|cc} 1 & 0 & 5 & 0 & 4 \\ 0 & 1 & 3 & -1 & 2 \end{array} \right)$$

$$a) \begin{cases} x_{11} + 5x_{31} = 0 \\ x_{21} + 3x_{31} = -1 \\ x_{31} = c_1 \end{cases}$$

$$\begin{cases} x_{11} = -5x_{31} \\ x_{21} = -3x_{31} - 1 \\ x_{31} = c_1 \end{cases}$$

$$b) \begin{cases} x_{12} + 5x_{32} = 4 \\ x_{22} + 3x_{32} = 2 \\ x_{32} = c_2 \end{cases}$$

$$\begin{cases} x_{12} = 4 - 5x_{32} \\ x_{22} = 2 - 3x_{32} \\ x_{32} = c_2 \end{cases}$$

$$X = \begin{pmatrix} -5c_1 & 4 - 5c_2 \\ -3c_1 - 1 & 2 - 3c_2 \\ c_1 & c_2 \end{pmatrix}, \quad c_1, c_2 \in \mathbb{R}$$



Домашнее задание

[П]: №№ 837, 839, 842, 845, 864.

Найти решение уравнения:

$$\begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & 1 & 3 & -1 \\ 1 & 2 & 4 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 0 & 1 \\ 1 & -1 & 2 \end{pmatrix}.$$

$$A \cdot X \cdot B = C \rightarrow X = A^{-1} C B^{-1}$$

$$1. \exists A^{-1}, B^{-1}$$

$$n \times n, m \times m$$

$$2. C, n \times m$$

$$X, n \times m$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \Delta = ad - bc$$

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

№ 863

$$A = \begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix}, B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}, C = \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}, A \cdot X \cdot B = C$$

$$|A| = -1 \neq 0$$

$$\exists A^{-1}$$

$$|B| = -2 \neq 0$$

$$\exists B^{-1}$$

$$\left(\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 5 & -2 & 0 & 1 \end{array} \right) \xrightarrow{-2I} \sim \left(\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ -1 & 0 & -2 & 1 \end{array} \right) \xrightarrow{+3I} \sim \left(\begin{array}{cc|cc} 0 & -1 & -5 & 3 \\ -1 & 0 & -2 & 1 \end{array} \right) \xrightarrow{\uparrow} \sim \left(\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & -1 & 5 & -3 \end{array} \right)$$

A

A⁻¹

$$B^{-1} = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}^{-1} = \begin{pmatrix} -4 & 3 \\ 7 & -5 \\ 2 & 2 \end{pmatrix}$$

$$X = \underbrace{\begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}}_{\begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}} \cdot \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix} \cdot \begin{pmatrix} -4 & 3 \\ 7 & -5 \\ 2 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}}}$$