№ 310 (Матв.)

$x^2 y' = x^2 y^2 + xy + 1$

$y' = y^2 + \frac{1}{x} y + \frac{1}{x^2}$ (1)

1. $y_* = \frac{\alpha}{x} \to (1)$

$-\frac{\alpha}{x^2} = \frac{\alpha^2}{x^2} + \frac{\alpha}{x^2} + \frac{1}{x^2}$

$\alpha^2 + 2\alpha + 1 = 0$

$\alpha = -1 \qquad y_* = -\frac{1}{x}$

2. $y = z + y_* = z - \frac{1}{x} \qquad z = z(x)$

(*) $y = z - \frac{1}{x} \to (1)$

$z' + \frac{1}{x^2} = z^2 - \frac{2}{x} z + \frac{1}{x^2} + \frac{1}{x} z - \frac{1}{x^2} + \frac{1}{x^2}$

$z' = z^2 - \frac{1}{x} z$ (2)

$y' = a(x) y^2 + b(x) y + c(x)$

$y' = A y^2 + \frac{B}{x} y + \frac{C}{x^2}$

$y_* = \frac{\alpha}{x}$

3. 1) $z = 0$

2) $\frac{z'}{z^2} = 1 - \frac{1}{x} \cdot \frac{1}{z}$

$u = \frac{1}{z} \qquad z = \frac{1}{u}$

$u' = -\frac{z'}{z^2}$

$-u' = 1 - \frac{1}{x} \cdot u$

$u' = \frac{1}{x} u - 1$ (3)

a) $u' = \frac{1}{x} u$

$\frac{du}{u} = \frac{dx}{x}$

$\ln|u| = \ln|x| + \ln|C|$

$u = C \cdot x$

б) $u = C(x) x \to (3)$

$C'(x) \cdot x + C(x) = C(x) - 1$

$C'(x) = -\frac{1}{x}$,

$C(x) = -\ln|x| + \tilde{C}$

$u = \tilde{C} x - x \ln|x|$

$z = \frac{1}{x(C - \ln|x|)}$

$z = 0$

$z = \frac{1}{x(C - \ln|x|)}$

(*)

$y = -\frac{1}{x}$

$y = -\frac{1}{x} + \frac{1}{x(C - \ln|x|)}$

№307 (матвеев)

$$y' = y^2 - xy - x \qquad (1)$$

1. $y_* = ax + b$,

$$a = a^2x^2 + 2abx + b^2 - ax^2 - bx - x$$

$$(a - a^2)x^2 + (b - 2ab)x + a - b^2 = -x$$

$$\begin{cases} x^2: & a(1-a) = 0 \\ x: & b(1-2a) = -1 \\ x^0: & a - b^2 = 0 \end{cases}$$

$a = 1$, $b = 1$

$y_* = x + 1$

2. $\boxed{y = z + x + 1}$ (2) $\longrightarrow$ (1)

$$z' + 1 = z^2 + x^2 + 1 + 2zx + 2z + 2x - xz - x^2 - x - x$$

$$z' = z^2 + (x+2)z \qquad (3) \qquad 1)\ z = 0$$

2) $\boxed{u = \frac{1}{z}}$ $z = \frac{1}{u} \rightsquigarrow (3)$: $u' = -(x+2)u - 1$ (4)

a) $u' = -(x+2)u$

$\frac{du}{u} = -(x+2)dx$

$\ln|u| = -\frac{x^2}{2} - 2x + \ln|C|$

$u = C \cdot e^{-\left(\frac{x^2}{2} + 2x\right)}$

б) $u = C(x)\, e^{-\left(\frac{x^2}{2} + 2x\right)}$ (5) $\rightarrow$ (4)

$C'(x)\, e^{-\left(\frac{x^2}{2} + 2x\right)} = -1$

$C'(x) = -e^{\frac{x^2}{2} + 2x}$

$C(x) = -\int e^{\frac{x^2}{2} + 2x}\, dx + \tilde{C} \rightarrow (5)$

$u = C \cdot e^{-\left(\frac{x^2}{2} + 2x\right)} - e^{-\left(\frac{x^2}{2} + 2x\right)} \cdot \int e^{\frac{x^2}{2} + 2x}\, dx$

$u = \frac{1}{z}$

$$y = x + 1$$

$$y = \frac{e^{\frac{x^2}{2} + 2x}}{C - \int e^{\frac{x^2}{2} + 2x}\, dx} + x + 1$$

№ 308 (нач.)

$$xy' = y^2 - (2x+1)y + x^2 + 2x \qquad (1)$$

1. $y_* = ax + b \longrightarrow (1)$

$$ax = a^2x^2 + 2axb + b^2 - (2x+1)(ax+b) + x^2 + 2x$$

$$(2a - a^2)x^2 + (2a + 2b - 2ab)x - b^2 + b = x^2 + 2x$$

$$\begin{cases} x^2: & a(2-a) = 1 \\ x: & 2(a+b-ab) = 2 \\ x^0: & b(1-b) = 0 \end{cases}$$

$b = 0,\ a = 1$ $\qquad y_* = x$

2. $y = z + x \qquad (2) \longrightarrow (1)$

$$x(z' + 1) = (z+x)^2 - (2x+1)(z+x) + x^2 + 2x$$

$$xz' = z^2 - z$$

$$xz' = z(z-1) \qquad (3)$$

1) $z = 0$, $z = 1$

2) $\dfrac{dz}{z(z-1)} = \dfrac{dx}{x}$

$$\int \frac{dz}{z(z-1)} = \int \left(\frac{-1}{z} + \frac{1}{z-1}\right) dx = \ln|z-1| - \ln|z|$$

$$\ln|z-1| - \ln|z| = \ln|x| + \ln|C|$$

$$\frac{z-1}{z} = C \cdot x$$

$z = 0$

$\dfrac{z-1}{z} = C \cdot x \longrightarrow (2)$

$y = x$

$\dfrac{y-x-1}{y-x} = C \cdot x$

$$1 - \frac{1}{y-x} = C \cdot x$$

$$1 - Cx = \frac{1}{y-x}$$

$$y - x = \frac{1}{1-Cx}$$

$$y = \frac{1}{1-Cx} + x$$

Отв.

$\mathcal{D}/z$: ~ 169 | $y_* = ax + b$

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