Методы решения ОДУ 1-го порядка

(обзорное занятие)

№ 354 [м]

$$\left(\frac{x}{y}+1\right)dx+\left(\frac{x}{y}-1\right)dy=0 \qquad \text{тип - однрод.}$$

$$\frac{y}{x}=t \qquad y=tx \qquad dy=t\,dx+x\,dt$$

$$x^2\left(\frac{y^2}{x^2}-2\frac{y}{x}-1\right)=C$$

$$\underline{(y^2-2yx-x^2)=C}$$

$$\left(\frac{1}{t}+1\right)dx+\left(\frac{1}{t}-1\right)(t\,dx+x\,dt)=0$$

$$\left(\frac{1}{t}+1+1-t\right)dx+\left(\frac{1}{t}-1\right)x\,dt=0$$

$$\frac{1+2t-t^2}{t}dx+x\cdot\frac{1-t}{t}dt=0$$

$$(1+2t-t^2)dx+x(1-t)dt=0 \quad \Big/ \ \frac{1}{x\cdot(1+2t-t^2)}$$

$$\cancel{\boxed{x=0}}$$

$$-t^2+2t+1=0$$

$$t^2-2t-1=0$$

$$t_{1,2}=1\pm\sqrt{2}$$

$$\frac{dx}{x}=+\frac{(1-t)\,dt}{t^2-2t-1}$$

$$\ln|x|=-\frac{1}{2}\ln|t^2-2t-1|+\frac{1}{2}\ln|c|$$

$$2\ln|x|=-\ln|t^2-2t-1|+\ln|c|$$

$$x^2=\frac{c}{t^2-2t-1} \qquad x^2(t^2-2t-1)=C, \quad x\neq 0$$

$$\left(\frac{x}{y}+1\right)dx + \left(\frac{x}{y}-1\right)dy = 0$$

$\frac{x}{y} = t$ $\quad x = ty$

$y \neq 0$

$$(t+1)(t\,dy + y\,dt) + (t-1)\,dy = 0$$

$$\big(t(t+1) + t - 1\big)\,dy + y(t+1)\,dt = 0$$

$$(t^2+2t-1)\,dy + y(t+1)\,dt = 0$$

№ 195

$$\underbrace{(x^2+y^2+x)}_{P}dx + \underbrace{y}_{Q}dy = 0$$

1. УПД ?

$$\frac{\partial P}{\partial y} = 2y \neq \frac{\partial Q}{\partial x} = 0$$

2. $\mu = \mu(x)$ $$\frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q} = \frac{2y}{y} = 2 = f(x)$$

$$\mu(x) = C \cdot e^{\int 2dx} = C \cdot e^{2x}, \quad \exists\, C = 1$$

$$\mu(x) = e^{2x}$$

$$\underbrace{e^{2x}(x^2+y^2+x)}_{P}dx + \underbrace{e^{2x}y}_{Q}dy = 0 \quad (*)$$

$$\frac{\partial P}{\partial y} = 2ye^{2x} \equiv \frac{\partial Q}{\partial x} = 2ye^{2x}$$

3. $u:$ $$\begin{cases} \frac{\partial u}{\partial x} = e^{2x}(x^2+y^2+x) \\ \frac{\partial u}{\partial y} = e^{2x}y \end{cases}$$

$$\frac{\partial u}{\partial y} = e^{2x}y \to u = \frac{e^{2x}y^2}{2} + \varphi(x)$$

$$e^{2x}y^2 + \varphi'(x) = e^{2x}(x^2+y^2+x)$$

$$\varphi'(x) = e^{2x}(x^2+x)$$

$$\varphi(x) = \int e^{2x}(x^2+x)\,dx + C_0$$

$$\frac{1}{2}e^{2x}(x^2+x) - \frac{1}{2}\int e^{2x}(2x+1)dx =$$

$$= \frac{1}{2}e^{2x}(x^2+x) - \frac{1}{2}\left(\frac{1}{2}e^{2x}(2x+1) - \int e^{2x}dx\right) =$$

$$= \frac{1}{2}e^{2x}\left(x^2+x - \frac{1}{2}(2x+1) + \frac{1}{2}\right) = \frac{1}{2}e^{2x}x^2$$

$C_0 = 0$

$$\frac{1}{2}e^{2x}(y^2+x^2) = C$$

$$e^{2x}(y^2+x^2) = C$$

$$(x^2+y^2+x)\,dx + y\,dy = 0$$

$$(x^2+y^2)\,dx + \underbrace{x\,dx + y\,dy}_{\frac{1}{2}dx^2 + \frac{1}{2}dy^2} = 0$$

$$(x^2+y^2)\,dx + \frac{1}{2}d(x^2+y^2) = 0$$

$x^2+y^2 = t$, $\quad t \neq 0$

$$t\,dx + \frac{1}{2}dt = 0$$

$$2\,dx = -\frac{dt}{t}$$

$$2x = -\ln t + \ln|C|$$

$$e^{2x} = \frac{C}{t}$$

$$t\,e^{2x} = C$$

$$\boxed{(x^2+y^2)e^{2x} = C}$$

$y^2 \qquad y\,dy = \frac{1}{2}d(y^2)$

$\boxed{y^2 = t}$

$$(x^2+t+x)\,dx + \frac{1}{2}dt = 0$$

$$2(x^2+t+x) + \frac{dt}{dx} = 0$$

$$\frac{dt}{dx} = -2(x^2+x) - 2t$$

1) $\frac{dt}{dx} = -2t \qquad t = C\cdot e^{-2x}$

2) $t = C(x)e^{-2x}$

$$C'(x)e^{-2x} - 2C(x)e^{-2x} = -2(x^2+x) - 2C(x)e^{-2x}$$

$$C'(x) = -2(x^2+x)e^{2x}$$

$$C(x) = -2\int e^{2x}(x^2+x)\,dx + \tilde{C} =$$

$$= -2\left(\frac{1}{2}x^2e^{2x}\right) + \tilde{C} = -x^2e^{2x} + \tilde{C}$$

$t = \tilde{C}e^{-2x} - x^2$

$y^2 = Ce^{-2x} - x^2$

$y^2 + x^2 = Ce^{-2x}$

№ 332

$$(xy^4 - x)\,dx + (y + xy)\,dy = 0$$

1) $$x(y^4-1)\,dx + y(x+1)\,dy = 0 \quad \Big/ \; \frac{1}{(x+1)(y^4-1)}$$

$x+1=0 \quad \underline{x=-1}$

$y^4-1=0$

$(y^2+1)(y^2-1)=0 \quad \underline{y=\pm 1}$

$$\underline{\frac{x\,dx}{x+1} = -\frac{y\,dy}{y^4-1}}$$

$$\int \frac{x\,dx}{x+1} = \int \frac{x+1-1}{x+1}\,dx = \int\left(1 - \frac{1}{x+1}\right)dx = x - \ln|x+1|$$

$$\int \frac{y\,dy}{y^4-1} = \begin{bmatrix} t = y^2 \\ dt = 2y\,dy \end{bmatrix} = \frac{1}{2}\int \frac{dt}{t^2-1} = \frac{1}{2}\left(\frac{1}{2}\ln|t-1| - \frac{1}{2}\ln|t+1|\right) = \frac{1}{4}\left(\ln|y^2-1| - \ln(y^2+1)\right)$$

$$\frac{y}{(y^2+1)(y-1)(y+1)} = \frac{Ay+B}{y^2+1} + \frac{C}{y-1} + \frac{D}{y+1}$$

$$\frac{1}{t^2-1} = \frac{A}{t-1} + \frac{B}{t+1} = \frac{1}{2}\,\frac{t+1-t+1}{t^2-1} = \underline{\frac{1}{2}\,\frac{1}{t-1} - \frac{1}{2}\,\frac{1}{t+1}}$$

$$4(x - \ln|x+1|) = \ln(y^2+1) - \ln|y^2-1| + C$$

$x = -1$

$y = \pm 1$