№ 1

$$\underbrace{\left(x^2\cos(x+y)+y\right)}_{P}dx+\underbrace{\left(x^2\cos(x+y)-x\right)}_{Q}dy=0 \qquad (1)$$

1.

$$\frac{\partial P}{\partial y}=-x^2\sin(x+y)+1$$

$$\frac{\partial Q}{\partial x}=+2x\cos(x+y)-x^2\sin(x+y)-1$$

2.

$$\mu=\mu(x)$$

$$\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}=f(x)$$

$$\frac{2-2x\cos(x+y)}{x\left(x\cos(x+y)-1\right)}=\frac{-2}{x}=f(x)$$

$$\mu(x)=C\cdot e^{\int f(x)\,dx}=\frac{C}{x^2}, \quad \exists\, C=1 \qquad \mu=\frac{1}{x^2}$$

3. $x=0$ – реш.

4. $(1)\cdot\mu(x)$

$$\left(\underbrace{\cos(x+y)+\frac{y}{x^2}}_{P}\right)dx+\left(\underbrace{\cos(x+y)-\frac{1}{x}}_{Q}\right)dy=0$$

УПД

$$\frac{\partial P}{\partial y}=-\sin(x+y)+\frac{1}{x^2}\equiv\frac{\partial Q}{\partial x}=-\sin(x+y)+\frac{1}{x^2}$$

$$u(x,y)=?\quad \begin{cases}\dfrac{\partial u}{\partial x}=\cos(x+y)+\dfrac{y}{x^2}\\[2mm] \dfrac{\partial u}{\partial y}=\cos(x+y)-\dfrac{1}{x}\end{cases}$$

$$u=\sin(x+y)-\frac{y}{x}+\varphi(x)$$

$$u = \sin(x+y) - \frac{y}{x} + \varphi(x) \leftarrow \exists C_0 = 0$$

$$\cos(x+y) + \frac{y}{x^2} + \varphi'(x) = \cos(x+y) + \frac{y}{x^2} \qquad \varphi'(x) = 0 \qquad \varphi(x) = C_0$$

$$\boxed{\sin(x+y) - \frac{y}{x} = C} \; ; \quad \underline{x = 0}$$

$$x\,dy + y\,dx = d(xy)$$

$$\left(\cos(x+y) + \frac{y}{x^2}\right)dx + \left(\cos(x+y) - \frac{1}{x}\right)dy = 0$$

$$\cos(x+y)\underbrace{(dx + dy)}_{d(x+y)} + \underbrace{\frac{y\,dx - x\,dy}{x^2}}_{d\left(\frac{y}{x}\right)} = 0$$

$$\underbrace{\cos(x+y)\,d(x+y)}_{} + d\left(\frac{-y}{x}\right) = 0$$

$$d(\sin(x+y)) - d\left(\frac{y}{x}\right) = 0 \qquad d\left(\sin(x+y) - \frac{y}{x}\right) = 0$$

$$\boxed{\sin(x+y) - \frac{y}{x} = C}$$

№ 2

$$\big(\underbrace{tg(x+y)+x}_{P}\big)dx+\underbrace{x}_{Q}dy=0$$

1. $\frac{\partial P}{\partial y}=\frac{1}{\cos^2(x+y)}\neq\frac{\partial Q}{\partial x}=1$

2. $\mu=\mu(x)$ $\quad\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}=\frac{\frac{1}{\cos^2(x+y)}-1}{x}\neq f(x)$

$\mu=\mu(y)$ $\quad\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{-P}\neq f(y)$

3. $\mu(x+y)=?$ $\qquad\mu=\mu(\overset{x+y}{\overset{\|}{\omega}})$

$$P\cdot\frac{\partial\mu}{\partial y}-Q\frac{\partial\mu}{\partial x}=\mu\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)$$

$$P\cdot\frac{d\mu}{d\omega}\cdot\underset{\underset{1}{\|}}{(\omega)'_y}-Q\frac{d\mu}{d\omega}\cdot\underset{\underset{1}{\|}}{\underbrace{(\omega)'_x}}=\mu\cdot\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)$$

$$(P-Q)\frac{d\mu}{d\omega}=\mu\cdot\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)$$

$$\frac{d\mu}{d\omega}=\mu_{(\omega)}\cdot\frac{\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}}{P-Q}=f(\omega)\overset{?}{=}f(x+y)$$

$$\frac{d\mu}{d\omega}=\mu(\omega)f(\omega)$$

$$\mu(\omega)=C\cdot e^{\int f(\omega)d\omega}$$

$$\frac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}{P - Q} = \frac{1 - \frac{1}{\cos^2(x+y)}}{tg(x+y)} = \frac{-tg^2(x+y)}{tg(x+y)} = -tg(x+y) = f(x+y)$$

$$1 + tg^2\alpha = \frac{1}{\cos^2\alpha}$$

$$\mu(w) = C \cdot e^{-\int tg\, w\, dw} = C \cdot e^{\ln|\cos w|} = C \cdot |\cos w|$$

$x + y$

$\mu(w) = \cos w$, $\quad \mu(x+y) = \cos(x+y)$, $\quad \cos(x+y) \neq 0$

$$\underbrace{(\sin(x+y) + x \cdot \cos(x+y))}_{P}\,dx + \underbrace{x \cdot \cos(x+y)}_{Q}\,dy = 0$$

УПД

$$\frac{\partial P}{\partial y} = \cos(x+y) - x\sin(x+y) = \frac{\partial Q}{\partial x} = \cos(x+y) - x\sin(x+y)$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = \sin(x+y) + x\cos(x+y) \\[2mm] \dfrac{\partial u}{\partial y} = x\cos(x+y) \end{cases}$$

$$\sin(x+y) + x\cos(x+y) + \varphi'(x) = \sin(x+y) + x\cos(x+y)$$

$$\downarrow$$

$$\varphi'(x) = 0$$

$$\varphi(x) = C_0$$

$$\frac{\partial u}{\partial y} = x\cdot\cos(x+y) \xrightarrow{\int,\, y} u = x\cdot\sin(x+y) + \varphi(x)$$

$$\exists C_0 = 0$$

$$\downarrow$$

$$\underline{x\sin(x+y) = C}$$

$$(\operatorname{tg}(x+y)+x)\,dx + x\,dy = 0$$

$$\operatorname{tg}(x+y)\,dx + x(dx+dy) = 0$$

$$\operatorname{tg}(x+y)\,dx + x\,d(x+y) = 0$$

$x+y=t$ $$\underline{\operatorname{tg} t\,dx + x\,dt = 0} \longrightarrow x\cdot\sin t = C \xrightarrow{t=x+y} \boxed{x\cdot\sin(x+y)=C}$$

$$-\operatorname{tg} t\,dx = +x\,dt$$

$x=0$ – реш.

$\operatorname{tg} t = 0 \iff \sin t = 0 \quad t=\pi k,\ k\in\mathbb{Z}$

$$\frac{dx}{x} = -\frac{dt}{\operatorname{tg} t}$$

$$\ln|x| = -\underbrace{\int\frac{\cos t}{\sin t}\,dt}_{-\ln|\sin t|} + \ln|C|$$

$$\ln|x| + \ln|\sin t| = \ln|C|$$

$$x\cdot\sin t = C$$

№ 357 : [М.]

$$(xy^2 + y)\,dx - x\,dy = 0$$

№ 362 [М] :

$$\left(2y + \frac{1}{(x+y)^2}\right)dx + \left(3y + x + \frac{1}{(x+y)^2}\right)dy = 0$$