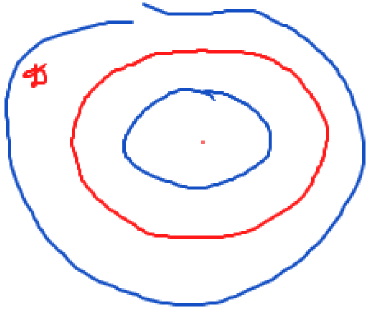
Уравнения в полных дифференциалах



$$P(x,y)\,dx + Q(x,y)\,dy = 0 \qquad (1)$$

$$\exists \quad u(x,y): \qquad du = P(x,y)\,dx + Q(x,y)\,dy$$

$$du = 0$$

$$\boxed{u(x,y) = C} \qquad (2)$$

$$D, \qquad \frac{\partial P}{\partial y} \equiv \frac{\partial Q}{\partial x} \qquad (3)$$

$$u(x,y): \qquad \begin{cases} \dfrac{\partial u}{\partial x} = P(x,y) \\[2mm] \dfrac{\partial u}{\partial y} = Q(x,y) \end{cases}$$

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(1) $2xy\,dx + (x^2 - y^2)\,dy = 0$

$P(x,y) = 2xy$, $\quad Q(x,y) = x^2 - y^2$

$\dfrac{\partial P}{\partial y} = 2x \quad \equiv \quad \dfrac{\partial Q}{\partial x} = 2x$ $\qquad$ УПД $\qquad \exists\ u(x,y)$

$du = \dfrac{\partial u}{\partial x}dx + \dfrac{\partial u}{\partial y}dy$

(2) $\begin{cases} \dfrac{\partial u}{\partial x} = 2xy \\ \dfrac{\partial u}{\partial y} = x^2 - y^2 \end{cases}$

$u = \int 2xy\,dx + \varphi(y)$

$u = yx^2 + \varphi(y) \quad (*) \xrightarrow{(2_2)}$

$x^2 + \varphi'(y) = x^2 - y^2$.

$\varphi'(y) = -y^2$

$\varphi(y) = -\dfrac{y^3}{3} + C_0 \rightarrow (*)$

$u(x,y) = yx^2 - \dfrac{y^3}{3} + C_0 \qquad$ Ⅎ $C_0 = 0$

$$\boxed{yx^2 - \dfrac{y^3}{3} = C}$$

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$$\frac{3x^2+y^2}{y^2}\,dx - \frac{2x^3+5y}{y^3}\,dy = 0 \ , \quad y \neq 0$$

1. $P(x,y) = \frac{3x^2+y^2}{y^2}$, $Q(x,y) = -\frac{2x^3+5y}{y^3}$

$$\frac{\partial P}{\partial y} = \left(\frac{3x^2}{y^2}+1\right)'_y = -\frac{6x^2}{y^3}$$

$$\frac{\partial Q}{\partial x} = \left(-\frac{2x^3}{y^3}-\frac{5}{y^2}\right)'_x = -\frac{6x^2}{y^3}$$

$\rightarrow$ УПД

$\exists\, u$:

2. $$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{3x^2+y^2}{y^2} \\[2mm] \dfrac{\partial u}{\partial y} = -\dfrac{2x^3+5y}{y^3} \end{cases}$$

$\int dx$

1) $u = \int\left(\frac{3x^2}{y^2}+1\right)dx + \varphi(y)$

$u = \frac{x^3}{y^2} + x + \varphi(y)$

$$-\frac{2x^3}{y^3} + \varphi'(y) = -\frac{2x^3}{y^3} - \frac{5}{y^2} \ ,$$

$$\varphi'(y) = -\frac{5}{y^2}$$

$$\varphi(y) = \frac{5}{y} + C_0$$

$\exists\, C_0 = 0$

$$u(x,y) = \frac{x^3}{y^2} + x + \frac{5}{y}$$

$$\boxed{\frac{x^3}{y^2} + x + \frac{5}{y} = C}$$

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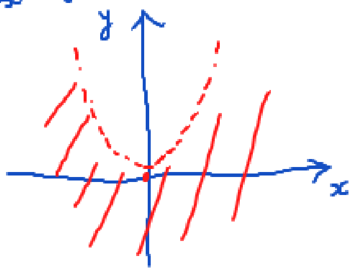
$$2x\left(1+\sqrt{x^2-y}\right)dx - \sqrt{x^2-y}\,dy = 0$$

$x^2-y>0$



1. $P(x,y) = 2x\left(1+\sqrt{x^2-y}\right)$

$Q(x,y) = -\sqrt{x^2-y}$

$$\frac{\partial P}{\partial y} = -\frac{x}{\sqrt{x^2-y}}$$
$$\frac{\partial Q}{\partial x} = -\frac{x}{\sqrt{x^2-y}}$$

$\rightarrow$ УПД, $\exists\, u$

2. $$\begin{cases} \dfrac{\partial u}{\partial x} = 2x\left(1+\sqrt{x^2-y}\right) \\ \dfrac{\partial u}{\partial y} = -\sqrt{x^2-y} \end{cases}$$

$$u = -\int\sqrt{x^2-y}\,dy + \varphi(x)$$
$$-\int\sqrt{x^2-y}\,dy = \int\sqrt{x^2-y}\,d(x^2-y)$$

(*) $$u = \frac{2}{3}\left(x^2-y\right)^{3/2} + \varphi(x)$$

$$\underline{2x\left(x^2-y\right)^{1/2}} + \varphi'(x) = 2x + \underline{2x\sqrt{x^2-y}}$$

$\varphi'(x) = 2x$, $\varphi(x) = x^2 + C_0$, $]\; C_0 = 0$

$$u(x,y) = \frac{2}{3}\left(x^2-y\right)^{3/2} + x^2$$

$$\boxed{\frac{2}{3}\left(x^2-y\right)^{3/2} + x^2 = C}$$

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$$\underbrace{\left(\frac{x}{\sin y} + 2\right)}_{P(x,y)} dx + \underbrace{\frac{(x^2+1)\cos y}{\cos 2y - 1}}_{Q(x,y)} dy = 0$$

$$\begin{cases} \sin y \neq 0 \\ \cos 2y - 1 \neq 0 \end{cases} \iff \sin y \neq 0$$

$$\cos^2 y - \sin^2 y - \sin^2 y - \cos^2 y = -2\sin^2 y$$

1. $$\frac{\partial P}{\partial y} = x\left(\frac{1}{\sin y}\right)'_y = -\frac{x \cdot \cos y}{\sin^2 y}$$

$$\frac{\partial Q}{\partial x} = \left(\frac{(x^2+1)\cos y}{\cos 2y - 1}\right)'_x = \frac{2x\cos y}{\cos 2y - 1} = -\frac{x\cos y}{\sin^2 y}$$

$\equiv \;\rightarrow$ УПД

$\exists\, u$

2. $$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{x}{\sin y} + 2 \\[2ex] \dfrac{\partial u}{\partial y} = \dfrac{(x^2+1)\cos y}{\cos 2y - 1} \end{cases}$$

$$\longrightarrow \quad u = \frac{x^2}{2\sin y} + 2x + \varphi(y)$$

$$\longrightarrow \quad -\frac{x^2\cos y}{2\sin^2 y} + \varphi'(y) = \frac{(x^2+1)\cos y}{\underbrace{\cos 2y - 1}_{-2\sin^2 y}}$$

$$\varphi'(y) = -\frac{\cos y}{2\sin^2 y}$$

$$\varphi(y) = -\frac{1}{2}\int \frac{\cos y\, dy}{\sin^2 y} + C_0$$

$$\varphi(y) = \frac{1}{2\sin y} + C_0, \quad C_0 \overset{?}{=} 0$$

$$u(x,y) = \frac{x^2}{2\sin y} + 2x + \frac{1}{2\sin y}$$

$$\boxed{\frac{x^2}{2\sin y} + 2x + \frac{1}{2\sin y} = C}$$

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$y' = 2 - y$

V $\quad y' + p(x)\cdot y = q(x)$

$$y' = \underbrace{q(x) - p(x)y}_{y\cdot\varphi(x)}$$

1) $y' + p(x)y = 0$

$y = C\cdot\tilde{y}$

2) $y = \underbrace{C(x)}\tilde{y}$

$C'(x) = \ldots;\quad C(x)$