$\cos z = A \qquad -1 \le A \le 1$

$$\Updownarrow$$

$$\begin{cases} \cos x = A \\ y = 0 \end{cases}$$

$\cos z = A \iff e^{iz} + e^{-iz} = 2A$

$\exists\ e^{iz} = u \qquad u + \frac{1}{u} = 2A$

$u^2 - 2Au + 1 = 0$

$D = 4A^2 - 4 = 4(A^2 - 1)$

а) $A = \pm 1, \quad D = 0$

$u^2 \pm 2u + 1 = 0 \qquad (u \pm 1)^2 = 0 \qquad u = \pm 1$

$e^{iz} = \pm 1$

1) $e^{iz} = 1$

$iz = \ln 1 + i(0 + 2\pi k),$

$iz = i\,2\pi k, \quad z = 2\pi k$

2) $e^{iz} = -1$

$iz = \ln|-1| + i(+\pi + 2\pi k)$

$z = (2k+1)\pi$

б) $-1 < A < 1, \quad D < 0 \qquad \sqrt{D} = 2\sqrt{A^2-1} = 2i\sqrt{1-A^2}$

$u_{1,2} = \dfrac{2A \pm 2i\sqrt{1-A^2}}{2} = A \pm i\sqrt{1-A^2}$

$|u_{1,2}| = \sqrt{A^2 + 1 - A^2} = 1$

$u_1 \cdot u_2 = 1$

1) $e^{iz} = A + i\sqrt{1-A^2}$

$iz = \ln|A + i\sqrt{1-A^2}| + i\left(\arg\left(A + i\sqrt{1-A^2}\right) + 2\pi k\right)$

$iz = i\left(\arg\left(A + i\sqrt{1-A^2}\right) + 2\pi k\right)$

$z = \underbrace{\arg\left(A + i\sqrt{1-A^2}\right)}_{\arccos A} + 2\pi k$

2) $e^{iz} = A - i\sqrt{1-A^2} \qquad u_2 = \overline{u_1}$

$\arg u_2 = -\arg u_1$

$z = -\arg\left(A + i\sqrt{1-A^2}\right) + 2\pi k$

$$\cos z = A, \quad A \in \mathbb{C}$$
$$\Updownarrow$$
$$e^{iz} + e^{-iz} = 2A, \quad \begin{cases} u = e^{iz} \\ u + \frac{1}{u} = 2A \end{cases}$$

$$u^2 - 2Au + 1 = 0, \quad D = 4A^2 - 4 = 4(A^2-1)$$

$$u_{1,2} = \frac{2A \pm 2\sqrt{A^2-1}}{2} = A \pm \sqrt{A^2-1}$$

$$u_1 = A + \sqrt{A^2-1}, \quad u_2 = A - \sqrt{A^2-1}$$

$$u_1 = A + i\sqrt{1-A^2}, \quad u_2 = A - i\sqrt{1-A^2}$$

$$u_1 \cdot u_2 = 1, \quad |u_1|\cdot|u_2| = 1, \quad \boxed{|u_2| = \frac{1}{|u_1|}} \quad (*)$$

$$u_2 = \frac{1}{u_1} = \frac{e^0}{|u_1|\cdot e^{i\arg u_1}} = \frac{1}{|u_1|}\cdot e^{-i\arg u_1}$$
$$\parallel$$
$$|u_2|\cdot e^{i\arg u_2} \qquad \boxed{\arg u_2 = -\arg u_1} \quad (**)$$

1) $e^{iz} = u_1$

$$iz = \ln|A + i\sqrt{1-A^2}| + i\left(\arg(A + i\sqrt{1-A^2}) + 2\pi k\right)$$
$$z = -i\ln|A + i\sqrt{1-A^2}| + \arg(A + i\sqrt{1-A^2}) + 2\pi k$$

2) $e^{iz} = u_2$

$$iz = \ln|A - i\sqrt{1-A^2}| + i\left(\arg(A \mp i\sqrt{1-A^2}) + 2\pi k\right)$$
$$z = -i\ln|A - i\sqrt{1-A^2}| + \arg(A - i\sqrt{1-A^2}) + 2\pi k$$
$$z \overset{*}{\underset{**}{=}} i\ln|A + i\sqrt{1-A^2}| - \arg(A + i\sqrt{1-A^2}) + 2\pi k$$

1) $\cup$ 2) $\boxed{z = \pm\left(-i\ln|A + i\sqrt{1-A^2}| + \arg(A + i\sqrt{1-A^2})\right) + 2\pi k}$ $k \in \mathbb{Z}$

$$\arccos A = -i\ln|A + i\sqrt{1-A^2}| + \arg(A + i\sqrt{1-A^2})$$
$$z = \pm\arccos A + 2\pi k, \quad k \in \mathbb{Z}$$

$\sin z = A, \qquad A \in \mathbb{C}$

$$\frac{e^{iz} - e^{-iz}}{2i} = A \qquad e^{iz} - e^{-iz} = 2Ai$$

$$\begin{cases} e^{iz} = u \\ u - \frac{1}{u} = 2iA \end{cases} \iff \sin z = A$$

$$u^2 - 2iAu - 1 = 0$$

$$D = -4A^2 + 4 = 4(1 - A^2), \qquad u_1 \cdot u_2 = -1, \quad \boxed{|u_2| = \frac{1}{|u_1|}}$$

$$u_2 = -\frac{1}{u_1},$$

$$|u_2| e^{i \arg u_2} = \frac{e^{\pi i}}{|u_1| \cdot e^{i \arg u_1}} =$$

$$= \frac{1}{|u_1|} \cdot e^{(\pi - \arg u_1) i}$$

$$\boxed{\arg u_2 = \pi - \arg u_1}$$

$$u_{1,2} = \frac{2iA \pm 2\sqrt{1-A^2}}{2} = iA \pm \sqrt{1-A^2}$$

1) $e^{iz} = u_1 = iA + \sqrt{1-A^2}$

$$iz = \ln|iA + \sqrt{1-A^2}| + i\left(\arg\left(iA + \sqrt{1-A^2}\right) + 2\pi k\right)$$

$$z = -i \ln|iA + \sqrt{1-A^2}| + \arg\left(iA + \sqrt{1-A^2}\right) + 2\pi k$$

2) $e^{iz} = u_2$

$$iz = \ln|u_2| + i(\arg u_2 + 2\pi k)$$

$$z = -i \ln \frac{1}{|u_1|} + (\pi - \arg u_1) + 2\pi k$$

$$z = i \ln|iA + \sqrt{1-A^2}| + \pi - \arg\left(iA + \sqrt{1-A^2}\right) + 2\pi k =$$

$$= -\left(-i \ln|iA + \sqrt{1-A^2}| + \arg\left(iA + \sqrt{1-A^2}\right)\right) + (2k+1)\pi$$

1) ∪ 2) $\Rightarrow$

$$\boxed{z = (-1)^n \left(-i \ln|iA + \sqrt{1-A^2}| + \arg\left(iA + \sqrt{1-A^2}\right)\right) + n\pi}$$

$$z = (-1)^n \arcsin A + n\pi$$